The Elasticity Pricing Rule for Two-sided Markets: A Note

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Abstract

Rochet and Tirole have derived an elasticity rule for relative prices in two-sided markets. This rule is seen as counterintuitive because it seems to imply that the “more elastic side of the market” is charged more. In this note it is argued that this interpretation is based on the assumption that elasticity of demand can be treated as a parameter. If elasticity is treated as function of price, the Rochet-Tirole rule is perfectly in line with economic intuition.

1 Introduction

Recent years have witnessed a rising interest in the theory of two sided-markets. It is by now well understood that two-sided markets differ in many respects from the standard one-sided market of most economic models (Wright, 2004). One much quoted result of Rochet and Tirole (2003) concerns the pricing behaviour of a monopolist platform provider. They are analyzing a market with two distinct groups of customers, “buyers” and “sellers”. Usage of the platform is the product of the two demands on the two sides of the market. Assuming log concave demand functions they find that the price structure \( p_B/p_S \) is given by

\[
\frac{p_B}{p_S} = \frac{\eta_B}{\eta_S}
\]

Thus, the relative price for one side of the market is positively related to elasticity – not negatively as in the standard Lerner formula. Rochet and Tirole (2006, p.655) can show, however, that the following formulas, reminiscent of the Lerner index apply:

\[
\frac{p_B}{p_S} = \frac{\eta_B}{\eta_S}
\]

where \( p_B \) and \( p_S \) are buyers’ and seller’ prices and \( \eta_B \) and \( \eta_S \) are the corresponding price elasticities.

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1 In the case of linear demand functions, equation (1) also describes the optimal price structure in the case of competing platforms or competing associations. Moreover, given linear demand, this price structure is Ramsey optimal. See Rochet and Tirole (2003, p.1009).
\[
(2) \quad \frac{p^b - (c - p^s)}{p^b} = \frac{1}{\eta^b} \quad \text{and}\]
\[
(3) \quad \frac{p^s - (c - p^b)}{p^s} = \frac{1}{\eta^s}
\]

These formulas are quite similar to the Lerner-index – except that costs have been replaced by opportunity costs; that is, costs are corrected by revenues extracted from the other side of the market.

2 The Rochet-Tirole result in the literature

Still, even when taking equations (2) and (3) into account, equation (1) remains valid and a number of economists have pointed out that Rochet and Tirole have derived a rather unusual result which is economically counterintuitive because it seems to imply that the more elastic group of customers will be charged more. For instance, Bolt and Tieman (2004, p.8) conclude that the result implies that “buyers pay a higher price than sellers” – given their assumption that buyers are more price elastic than sellers. As they point out, this result is difficult to reconcile with economic intuition (Bolt and Tieman 2004, p.8):

“Interior pricing in two-sided markets means that the most price-elastic side of the market pays the highest price, while standard economic intuition would predict the opposite.”

Moreover, they are stressing that the elasticity pricing rule can neither “easily explain the widely observed skewness in pricing towards one side of the market” nor the fact that price elastic consumers are charged less than relatively inelastic merchants (Bolt and Tieman 2004, p.8). Therefore, they conclude that log concavity of demand functions may not be an appropriate assumption.

Other examples are DeGrauwe, Rinaldi and Van Cayseele (2006, p.50) who state that

“If merchants are inelastic and cardholders are elastic, then the first group will face high charges and the latter will pay low prices.”

and Roson (2005, 148) who points out that:

“Interestingly, prices applied to the two market sides are both directly proportional to the price elasticity (\(\varepsilon\)) of the corresponding demand. In other words, an increase in the elasticity in one sub-market increases the specific relative price.”

DeGrauwe, Rinaldi and Van Cayseele (2006, pp.51-2) try to provide an intuition. Under the assumption that merchants are more elastic, they try to explain why merchants should be charged more:

“When the platform increases its charges from 1 to 2 euro to the merchant side, which is elastic, many merchants will drop out. In order to compensate for the reduced number of merchants, the platform will need to make the service more attractive, by attracting more cardholders. The latter are inelastic, so a large decrease in the price they need to pay will be needed to do the job. Or a 1 euro

\[\text{Note that most authors, including Bolt and Tieman quoted above, assume that merchants are less elastic than consumers.}\]
increase in the merchant price (from 1 to 2, or 100 percent) is matched by a substantial decrease in the charges to the cardholders.”

However, they fail to see, that in such a case it would seem much better to decrease the price for merchants, triggering a large increase in merchant demand, and increase the price for consumers, triggering only a small reduction in consumer demand. Thus, it remains obscure why the less elastic group (consumers in their example) should be charged less.

Bolt and Tieman (2004) do not try to explain what drives this counterintuitive result. Rather, they propose to use different demand functions. Assuming constant elasticity demand, they show that a corner solution will be profit maximising. This corner solution implies a pricing structure that is more in line with intuition: high prices for the less elastic market side and low prices for the elastic market side.

3 The role of elasticity

As the examples above show, economists have had problems to properly explain the Rochet-Tirole result. On close reflection, however, the result of Rochet and Tirole is not counterintuitive, at all. The economists quoted above do not sufficiently take into account that elasticity is a function of price and that equation (1) only provides an implicit solution of the optimal price structure.

When speaking about “the elasticity” of merchants and cardholders, they seem to have the slopes of inverse demand curves in mind. Steep slopes are identified with “inelastic demand” and flat slopes are identified with “elastic demand”.

Bolt and Tieman (2004, 7), for instance, define a higher price-elasticity of buyers in the following way:

\[
\eta^b(p) \geq \eta^s(p) \text{ for every feasible fee } p \geq 0
\]

This definition, however, is not useful in the present context because buyers and sellers can be charged separate prices. So, even if buyers’ elasticity is higher than sellers’ elasticity for any uniform price, this need not be the case if prices are differentiated. Indeed, high elasticity may be the result of a high price – not the cause. Similarly, low elasticity may be caused by low prices. As the discussion of the example of DeGrauwe, Rinaldi and Van Cayseele (2006) already showed, it seems much more plausible to charge a high price to the “inelastic” side of the market.

In the standard case with linear demand \(D\), elasticity \(\eta\) is driven by the ratio of \(p\) and \(D\) which can vary between zero and infinity. The positive relationship between price and elasticity is not confined to the linear case. As can be shown, this relationship generally holds for log concave demand functions. Taking the first derivative of the elasticity with respect to price yields:

\[
\frac{d\eta}{dp} = -\frac{D^p}{D} \frac{D^\prime}{D} + \frac{(D^\prime)^2 p}{D^2} > 0
\]
Since log concavity implies that $D'^D < (D')^2$, the derivative of elasticity with respect to price must be positive. Thus, for the group of log concave demand functions, equation (1) cannot be interpreted as a simple price setting rule for service providers in two-sided markets.\textsuperscript{3}

Figure 1 shows that it is indeed more profitable to raise prices for the relatively inelastic group and lower them for relatively elastic group. Initially, sellers and buyers are charged the same price ($P_0^S = P_0^B$). An increase of the seller's price by $\Delta$ and a corresponding decrease of the buyer's price (keeping $P = P^S + P^B$ constant) reduces $D^S$ by a small amount and increases $D^B$ by a large amount. If, conversely, $P^B$ is increased by $\Delta$ and $P^S$ is lowered by $\Delta$, the effect is a large decrease in $D^B$ and a small increase in $D^S$. Thus, from the point of view of the platform provider it is preferable to apply strategy 1—to raise prices for the relatively inelastic side of the market and to lower it for the elastic side. In accordance with the Rochet/Tirole rule, prices are adjusted to a point where $D^S$ is relatively elastic and $D^B$ is relatively inelastic (in line with equation 1).

![Figure 1: An example with “inelastic” sellers and “elastic” buyers](image)

$D^S = \text{demand of buyers and sellers}; \ P^{S/B} = \text{prices for buyers and sellers}; \ \text{subscripts denoting the initial position (0), strategy 1 or strategy 2.}$

As can be shown with the help of a few numerical examples, the result derived above applies to various combinations of demand curves. Platform providers will raise prices in

\textsuperscript{3} See also Weyl (2008, p.19) who points out that equation (1) only holds in equilibrium.
those markets conventionally understood to be inelastic and lower prices in those markets conventionally understood to be elastic (see Figure 2). The price for the group with a steep inverse demand curve is driven up until the elasticity of demand is high. Similarly, the price of the group with a flat demand curve is lowered to the point where demand is inelastic. The ‘elastic’ group will pay a higher price only in the case where the relatively flat curve lies far above and to the right of the steep curve (case 2):

Figure 2: Optimal prices in a two-sided market: four examples

Note: \( c = \text{constant marginal costs; } D^{S/B} = \text{demand of buyers and sellers; optimal monopoly prices are indicated by green points. Optimal total price } p = p^S + p^B; \text{ in cases } 1,3,4: \$6, \text{ in case } 2: \$7 (considering only integers).}

Further analysis of the linear case yields the result that the structure of prices can be traced back to the differences between maximum reservation prices. Given two demand curves as indicated by equation (6):

\[
(6) \quad D^{B/S} = a^{B/S} - b^{B/S}p^{B/S}
\]
and using equation (1) we can write

\[(7) \quad p^B - p^S = a^B/b^B - a^S/b^S\]

where \(a^B/b^B\) and \(a^S/b^S\) are the maximum reservation prices of buyers and sellers respectively.

Thus, in the linear case, reservation prices are the decisive factor in explaining relative prices of the two market sides. This result is in line with Weyl (2006) who introduces the concept of “vulnerability” of demand.\(^4\) “Vulnerability” is defined as an elasticity-weighted price (see equation 8).

\[(8) \quad \gamma(p) = \frac{p}{\eta} = -\frac{D}{D'}\]

In the linear case, vulnerability is simply equal to the difference between the reservation price and the market price.\(^5\)

\[(9) \quad \gamma(p) = \frac{a}{b} - p\]

Using the definition of vulnerability, Weyl (2006, p.9) re-formulates the profit maximizing condition of Rochet and Tirole (2003, 996):

\[(10) \quad p^S + p^B - c = \gamma^B(p^B) = \gamma^S(p^S)\]

Thus, for the profit maximizing monopolist vulnerability of demand must be equal on both sides of the market. It is straightforward to see that (9) in connection with (10) implies equation (7).

Moving from linear demand to a more general specification, the results has to be modified somewhat. Weyl (2006, 23) introduces a “linear vulnerability demand function”:

\[(11) \quad D(p) = \begin{cases} \frac{(a-p)^a}{b} & p \leq a \\ 0 & p > a \end{cases}\]

with \(0 < \alpha \leq 1\) and \(b > 0\).

In this case, vulnerability is given by (12):

\[\]

\(^{4}\) In Weyl (2008, p.8) he uses the term “market power” instead.

\(^{5}\) This difference is equal to two times the average consumer surplus.
(12) \[ \gamma(p) = \frac{a - p}{\alpha} \]

Inserting equation (12) into equation (10) yields:

\[ (13) \quad p^S + p^B - c = \frac{a^S - p^S}{\alpha^S} = \frac{a^B - p^B}{\alpha^B} \]

Equation (13) implies that the seller price (buyer price) is positively related with the reservation price \( a^S (a^B) \) – just as in the case of linear demand – and it is negatively related with \( \alpha^S (\alpha^B) \) which can be interpreted as a parameter reflecting the curvature of the demand curve. Thus, in line with intuition, the market side with a steeper demand curve (lower \( \alpha \)) will pay a higher price, ceteris paribus.

As the discussion above shows, log concave demand functions may yield the same results as the corner solution derived by Bolt and Tieman (2004). This is an important insight because it implies that we do not have to rely on convex demand functions. Rather, theoretically derived pricing rules and observed pricing behaviour in two-sided markets can be reconciled for a wide array of demand functions.

4 Concluding remarks

There is a long tradition of treating elasticities almost like constants. The Amoroso-Robinson relation and the Lerner index are prominent examples. However, as the discussion of the Rochet-Tirole result shows, such a treatment may be confusing. Therefore, whenever possible, prices (or price ratios) should be traced back to true underlying parameters. This paper has used linear demand curves and (following Weyl, 2006) “linear vulnerability demand curves” to show that relative prices in two-sided markets can be traced back to reservation prices and the curvatures of demand curves.

5 References


